Dimensionless Measures of Turbulent Magnetohydrodynamic Dissipation Rates

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ABSTRACT

The magnetic Reynolds number R_M, is defined as the product of a characteristic scale and associated flow speed divided by the microphysical magnetic diffusivity. For laminar flows, R_M also approximates the ratio of advective to dissipative terms in the total magnetic energy equation, but for turbulent flows this latter ratio depends on the energy spectra and approaches unity in a steady state. To generalize for flows of arbitrary spectra we define an effective magnetic dissipation number, $R_{M,e}$, as the ratio of the advection to microphysical dissipation terms in the total magnetic energy equation, incorporating the full spectrum of scales, arbitrary magnetic Prandtl numbers, and distinct pairs of inner and outer scales for magnetic and kinetic spectra. As expected, for a substantial parameter range $R_{M,e} \sim O(1) << R_M$. We also distinguish $R_{M,e}$ from $\tilde{R}_{M,e}$ where the latter is an effective magnetic Reynolds number for the mean magnetic field equation when a turbulent diffusivity is explicitly imposed as a closure. That $R_{M,e}$ and $\tilde{R}_{M,e}$ approach unity even if $R_M >> 1$ highlights that, just as in hydrodynamic turbulence, energy dissipation of large scale structures in turbulent flows via a cascade can be much faster than the dissipation of large scale structures in laminar flows. This illustrates that the rate of energy dissipation by magnetic reconnection is much faster in turbulent flows, and much less sensitive to microphysical reconnection rates compared to laminar flows.

1. Introduction

Magnetic fields play an observable and dynamical role in a range of astrophysical and laboratory plasmas. When the scales of interest are larger than the particle mean free path, magnetohydrodynamics provides a suitable approximation (Mofffat 1978, Parker 1979, Biskamp 1997). Derived from Maxwell's equations and Ohm's law, the induction equation describing the time evolution of the magnetic field $\mathbf{B}$ in non-relativistic magnetohydrodynamics (MHD) is given by

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \lambda \nabla^2 \mathbf{B}, \tag{1}$$

where λ is the magnetic diffusivity, and $\mathbf{v}$ is the velocity, determined from the fluid momentum equation. For incompressible flows, the momentum equation incorporating the Lorentz force is

$$\partial_t \mathbf{v} = -\mathbf{v} \cdot \nabla \mathbf{v} - \nabla P/\rho + (\nabla \times \mathbf{B}) \times \mathbf{B}/4\pi\rho + \nu \nabla^2 \mathbf{v}, \tag{2}$$

where P is the pressure and ν is the kinematic viscosity, and ρ is the density.

Taking the approximate ratio of magnitudes of the second to third terms in (1) introduces the magnetic Reynolds number

$$R_{M,l} \equiv lv/\lambda, \tag{3}$$

where l is a macroscopic scale of field variation for the system being studied. When l is the largest macroscopic scale of the system, we write R_M, eliminating the subscript l.

For laminar flows in which $R_M >> 1$, the last term of (1) is often ignored. An important consequence of ignoring this term is magnetic flux freezing, $d_t \int \mathbf{B} \cdot d\mathbf{S} = 0$ (Parker 1979) which implies that the integrated product of the magnetic field and the surface area it penetrates, or the magnetic flux moving with the plasma, is conserved. Dotting (1) with $\mathbf{B}$ gives the same R_M for the corresponding terms in the magnetic energy equation so hat $R_M >> 1$ would also imply that the resistive dissipation term is small compared to the other dynamical terms. When most of the magnetic energy is on the largest scale, the implication is that the dissipation of the total magnetic energy is negligible.

The above physical implications of $R_M >> 1$ do not apply if the flow is turbulent. When an initially laminar flow becomes turbulent, the field and flow evolve to develop non-trivial turbulent spectra (e.g. Biskamp 1997, Biskamp 2003). Eqn. (3), with l taken as the largest macroscopic scale of the system, is no longer a good approximation to the ratio of the advection to dissipation term in the total magnetic energy equation even if the energy is dominated by structures on that scale. The dissipation term is not necessarily

negligible because it depends more strongly on high wavenumber components of the turbulent spectrum than do the advection terms.

The importance of dissipation in MHD turbulence just described, is directly analogous to the case of hydrodynamic turbulence. For the latter, the Kolmogorov steady-state assumption (Kolmogorov 1941; see also Davidson 2004) that the energy dissipation rate per mass is constant on all scales through the cascade implies that the energy dissipation rate at the dissipation scales exactly equals that at the input scale. Therefore, the dissipation term in the total energy evolution equation exactly balances the nonlinear energy transfer terms when all scales are included and a steady-state is assumed. Extracting dimensionless measures of hydrodynamic turbulence dissipation rates have confirmed this basic conceptual picture (e.g. Pearson et al. 2004).

In making contact with these concepts from hydrodynamic turbulence, we find it particularly important to emphasize that dissipation is non-negligible in MHD turbulence even though $R_M >> 1$. This very much contrasts the physical implications commonly attributed to laminar $R_M >> 1$ MHD flows. Characterizing the MHD energy depletion time scales for large R_M turbulent MHD flows such as those of stellar convection zones, accretion disks, and galactic interstellar media, is important for understanding the magnetic field evolution, conversion of magnetic energy into particles, and requirements for magnetic field sustenance. Since these flows are characterized by a spectrum of scales (Biskamp 1997, Biskamp 2003), the ratio of advection to dissipation terms in the total magnetic energy equation is not in general R_M but a generalized magnetic dissipation number $R_{M,e}$ which depends on the magnetic and kinetic energy spectra.

Quantifying $R_{M,e}$, requires incorporating the spectrum of magnetic and velocity fields along with the magnetic Prandtl number $Pr_M \equiv \nu/\lambda$. Here we calculate $R_{M,e}$ as a function of the magnetic spectral index and Pr_M, the latter of which depends on the microphysical diffusivities and thus on R_M. Although R_M (as defined with the largest scale) does help determine whether the flow would have become turbulent in the first place, it is not a good indicator of the time scale for overall energy dissipation when the flow is turbulent. For regimes in which $R_{M,e}$ is largely independent of R_M, the total energy depletion rate becomes largely independent of the specific small scale mechanism of dissipation; turbulence cascades the energy to scales where the dissipation time is short.

In section 2, we derive the approximations for the corresponding second and third terms of (1) in the magnetic energy equation, incorporating the turbulent spectra and triple correlations for the case without a mean field. We then plot $R_{M,e}$ as a function of the magnetic spectral index and Pr_M. In section 3, we distinguish $R_{M,e}$ from a different quantity $\tilde{R}_{M,e}$ for the mean field equation. In section 4, we conclude with a discussion of

the implications of $\tilde{R}_{M,e} << 1$ and $R_{M,e} << R_M$.

2. $R_{M,e}$ in the absence of a mean field

We write the magnetic field as a sum of mean and fluctuating contributions such that $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$ where $\overline{\mathbf{B}}$ is the mean field that survives the averaging over scales much larger than the scale of any fluctuations incorporated in $\mathbf{b}$. We use the same approach for the velocity, but we will assume here that the mean velocity $\overline{\mathbf{V}} = 0$.

We first consider $\overline{\mathbf{B}} = 0$ for which equation for the total fluctuating magnetic field (written in Alfvén speed units), obtained by subtracting the average of (1) from (1), is

$$\partial_t \mathbf{b} = \nabla \times (\mathbf{v} \times \mathbf{b}) - \nabla \times \langle \mathbf{v} \times \mathbf{b} \rangle + \lambda \nabla^2 \mathbf{b}. \tag{4}$$

Dotting this equation with $\mathbf{b}$ and averaging, we obtain

$$\frac{1}{2}\partial_t \langle b^2 \rangle = \langle \mathbf{b} \cdot \nabla \times (\mathbf{v} \times \mathbf{b}) \rangle + \lambda \langle \mathbf{b} \cdot \nabla^2 \mathbf{b} \rangle. \tag{5}$$

The effective magnetic dissipation number $R_{M,e}$ represents an estimate of the ratio of magnitudes of the second to third term in (5).

When an initially laminar system is subject to a source of turbulence in the velocity field, both the velocity and magnetic field quickly evolve nonlinearly to acquire turbulent spectra. Even if the magnetic initial field were weak, the system typically evolves to a state of near equipartition between total kinetic and magnetic energies, but each with distinct spectra (Schekochihin et al. 2002, Schekochihin et al. 2004, Haugen et al. 2003, Haugen et al. 2004). The triple fluctuation terms in e.g (5) mediate the nonlinear sustenance of the turbulent spectra. To obtain quantitative estimates of (10) and (11), we write the magnitude of the magnetic field and velocity in terms of angle-integrated kinetic and magnetic energy spectra $E_M(k)$ and $E_V(k)$ respectively. We have

$$v(k) = (2kE_V(k))^{1/2} = v_0 \left(\frac{k}{k_{0,v}}\right)^{\frac{1-q}{2}} \tag{6}$$

and

$$b(k) = (2kE_M(k))^{1/2} = b_0 \left(\frac{k}{k_{0,b}}\right)^{\frac{1-m}{2}}, \tag{7}$$

where q and m are the kinetic and magnetic energy spectral indices, v_0 and b_0 are the magnitude of the velocity and magnetic field on the outer turbulent scale, and $k_{0,v}, k_{0,b}$ are the wave numbers for the outer scales of the assumed kinetic and magnetic spectra.

We also allow for different inner scales, or equivalently, allow the magnetic Prandtl number Pr_M to be a free parameter. To accommodate this, note that at the scale k_λ^{-1}, the magnetic spectrum is truncated by dissipation. We assume that the kinetic energy cascade is mediated by local interactions so that $\nu \sim v_\nu/k_\nu$, where v_ν and k_ν are the velocity and wave number of the viscous scale. There must also be a characteristic speed v_{eff} such that $\lambda = v_{eff}/k_\lambda$. However, the value v_{eff} need not be v_λ (the value of v at the scale k_λ^{-1}), and cannot be for $Pr_M >> 1$, since then $v_\lambda \sim 0$. Larger scale velocities fold the field which can show up as magnetic power on small scales (Schekochihin et al. 2002, Schekochihin et al. 2004, Haugen et al. 2003, Haugen et al. 2004). We then posit that $b_\lambda \sim v_{eff}$, since b_λ (the field at k_λ) draws its energy from v_{eff}. Using the definition of $Pr_M \equiv \frac{\nu}{\lambda}$, we then have

$$\lambda \simeq \frac{b_\lambda}{k_\lambda} \simeq \frac{1}{Pr_M}\frac{v_\nu}{k_\nu} \tag{8}$$

Eq. (8) implies

$$R_M \equiv \frac{v_0}{k_{0,v}\lambda} = Pr_M \left(\frac{k_\nu}{k_{0,b}}\right)^{\frac{q+1}{2}} \left(\frac{k_{0,b}}{k_{0,v}}\right)^{\frac{q+1}{2}} = \frac{v_0}{b_0}\left(\frac{k_\lambda}{k_{0,b}}\right)^{\frac{m+1}{2}}. \tag{9}$$

Assuming that the magnetic and kinetic spectra for $k \leq Min[k_\lambda, k_\nu]$ are mediated primarily by local interactions, we can approximate each nonlinear term on the right of (5) by an integral over the magnitude of k. For $k_{0,b} \geq k_{0,v}$, the second term of (5) gives

$$|\langle \mathbf{b}\cdot\nabla\times(\mathbf{v}\times\mathbf{b})\rangle| \lesssim v_0 b_0^2 \int_{k_{0,b}}^{k_d} \left(\frac{k}{k_{0,v}}\right)^{\frac{1-q}{2}} \left(\frac{k}{k_{0,b}}\right)^{1-m} dk = k_{0,b} v_0 b_0^2 \left(\frac{k_{0,b}}{k_{0,v}}\right)^{\frac{1-q}{2}} \int_1^{k_d/k_{0,b}} \kappa^{\frac{3-q-2m}{2}}\, d\kappa, \tag{10}$$

where $k_d/k_{0,b} \equiv Min[k_\nu/k_{0,b}, k_\lambda/k_{0,b}] = Min\left[\frac{k_{0,v}}{k_{0,b}}\left(\frac{R_M}{Pr_M}\right)^{\frac{2}{q+1}}, \left(\frac{R_M b_0}{v_0}\right)^{\frac{2}{m+1}}\right]$, using (9). The inequality in (10) arises because the left side includes the competing effects of turbulent amplification and turbulent diffusion of the magnetic field. The norm of the third term of (5), similarly averaged, is given by

$$|\lambda\langle \mathbf{b}\nabla^2\mathbf{b}\rangle| \sim Pr_M^{-1}\frac{v_\nu}{k_\nu} b_0^2 \int_{k_{0,b}}^{k_\lambda} k \left(\frac{k}{k_{0,b}}\right)^{1-m} dk = \frac{k_{0,b} v_0 b_0^2}{R_M}\left(\frac{k_{0,b}}{k_{0,v}}\right) \int_1^{k_\lambda/k_{0,b}} \kappa^{2-m} d\kappa, \tag{11}$$

where we have again used (9). Because (11) involves only dissipation, whereas the left side of (10) could involve amplification or diffusion, $R_{M,e}$, estimated by the ratio of the right side of (10) to the right side of (11), represents an upper limit to the ratio of the left hand sides of those equations.

To use the ratio of (10) to (11) to estimate $R_{M,e}$, we need values for q, m, b_0^2/v_0^2, R_M and Pr_M. We will assume that $q = 5/3$ and plot $R_{M,e}$ as a function of independent variables

m and Pr_M for various values of R_M. For $m > 1$, b_0^2/v_0^2 can be approximated by the ratio of total magnetic to kinetic energy because

$$\frac{b_0^2}{v_0^2} = \frac{E_M}{E_V}\frac{\int_1^{\frac{k_\nu}{k_{0,v}}} \kappa^{-5/3}d\kappa}{\int_1^{\frac{k_\lambda}{k_{0,b}}} \kappa^{-m}d\kappa} = \frac{E_M}{E_V}\frac{\int_1^{\frac{k_\nu}{k_{0,v}}} \kappa^{-5/3}d\kappa}{\int_1^{\frac{k_\lambda}{k_{0,b}}} \kappa^{-m}d\kappa} = \frac{3E_M}{2E_V}\frac{1-(R_M/Pr_M)^{-1/2}}{\frac{1}{m-1}\left(1-(R_M b_0/v_0)^{\frac{2-2m}{1+m}}\right)} \sim \frac{E_M}{E_V}, \tag{12}$$

where the similarity follows self-consistently for $R_M b_0/v_0, R_M/Pr_M >> 1$. For modest R_M and Pr_M, the same approximation is also satisfactory for $m = 1$, as the integrals in the second term of (12) become ratios of logarithms. In general, b_0^2 is of order v_0^2 for the range of m considered. Remember that b_0 and v_0 are defined at $k_{0,b}$ and $k_{0,v}$ respectively and the latter two wave numbers need not be equal: Although we assume m and q are constant over the ranges of $k_{b,0} \leq k \leq k_\lambda$ and $k_{v,0} \leq k \leq k_\nu$ respectively, the allowance of $k_{v,0} \neq k_{b,0}$ and $k_\lambda \neq k_\nu$ provides flexibility in incorporating a range of spectra compatible with MHD simulations (Schekochihin et al. 2002, Schekochihin et al. 2004, Haugen et al. 2003, Haugen et al. 2004).

Figs. 1 and 2 show $R_{M,e}$ calculated from the ratio of (10) to (11) for $10^{-2} \leq Pr_M \leq \frac{R_M}{10}$, and $1 \leq m \leq 1.7$ using two values of $k_{v,0}/k_{b,0}$ and two values of $(b_0/v_0)^2$. For Fig. 1, $R_M = 10^3$ and for Fig. 2 $R_M = 10^6$. To ensure $k_\nu >> k_{v,0}$, we must have $R_M/Pr_M >> 1$, which explains why we have chosen the upper limits on Pr_M to be $R_M/10$ in each plot.

The case of $(b_0/v_0)^2 = 1/4$ used in Fig. 1bd and 2bd is motivated by saturated states of non-helical MHD turbulence which show that in cases when the magnetic energy is initially weak, it builds up to a fraction of equipartition of the total kinetic energy when the latter is steadily driven (Schekochihin et al. 2004, Haugen et al. 2004, Haugen et al. 2003, Maron et al. 2004, Haugen et al. 2004). We have not required a steady state, but in that case the two terms on the right of (4) would exactly balance. This could be used to set the value of $(b_0/v_0)^2$ for a given m, $k_{v,0}/k_{b,0}$, Pr_M, and R_M, thereby enforcing $R_{M,e} = 1$. However, the scaling approximation for the first term on the right of (10) is crude, and does not take into account that terms within that term can have different signs, reducing its value and that of $R_{M,e}$. In the steady state, $R_{M,e}$, could therefore modestly exceed unity using our estimation procedure. As seen in Fig. 1 however, the important point is that $R_{M,e} << R_M$ robustly.

The dependencies of $R_{M,e}$ on m, R_M, Pr_M, $k_{v,0}/k_{b,0}$, and $(b_0/v_0)^2$ are evident in Figs. 1 and 2. In each panel, as m increases for a fixed R_M, the magnetic spectrum steepens and becomes more heavily weighted by low k for the same value of $(b_0/v_0)^2$, increasing $R_{M,e}$. A larger value of $(b_0/v_0)^2$ lowers $R_{M,e}$, as can be seen by comparing the two rows of Fig. 1. This is expected: for a given m, increasing $(b_0/v_0)^2$ increases the upper bound in (11) and that integral is more sensitive to the upper bound than the integral of (10). Since

$R_{M,e} \propto (k_{v,0}/k_{b,0})^{\frac{1+q}{2}}$, increasing $k_{v,0}/k_{b,0}$ also increases $R_{M,e}$ as evidenced by the figures.

Each panel of Fig. 1 shows that for a fixed m, increasing Pr_M lowers $R_{M,e}$. This is because more spectral range is available for the magnetic field at large k as Pr_M is increased, thereby increasing the relative weight of high k contributions of the field in the dissipation term. At large Pr_M and low m, $R_{M,e} \to 0$ since the shallow spectrum again increases the relative importance of high k for the dissipation term; the advection term is ruled by the $k_\nu << k_\lambda$ velocity cutoff and is less sensitive to increases in Pr_M for $Pr_M >> 1$.

Comparing Figs. 1 and 2, we see that increasing R_M increases, $R_{M,e}$ only weakly, highlighting the role of the spectra and the universality of $R_{M,e} << R_M$. We reiterate that $R_{M,e}$ is calculated from the ratio of the advection to microphysical dissipation terms, NOT by approximating any of the advection terms via a turbulent diffusivity as is commonly done in mean field formalism (Mofffat 1978, Zeldovich et al. 1983). We discuss the latter in the next section.

3. Effective dissipation number for the mean field equation

When the dynamical equations of a turbulent system such as an accretion disk (Shakura and Sunyaev 1973, Frank et al. 1992, Balbus and Hawley 1998) are assumed to obey a symmetry such as axisymmetry, these equations describe the evolution of mean quantities, not total quantities. Turbulence violates the global symmetry locally and only the mean quantities obey the symmetry. Accretion disk theory that invokes a turbulent viscosity (Shakura and Sunyaev 1973) in the axisymmetric velocity equation is in fact a mean field theory, even when not explicitly presented as such. The same is true for the mean magnetic field equation (Mofffat 1978, Zeldovich et al. 1983). The associated turbulent diffusivities are formally constructed from closures that replace mean nonlinear correlations of macroscopic turbulent fluctuations in the momentum and induction equations by diffusion terms, NOT from the replacement of microphysical dissipation terms. This is an important distinction from the previous section which involved no closures on the second term of (5); there we simply took the ratio of that term to the third term to define $R_{M,e}$.

More explicitly, consider the equation for evolution of the mean magnetic field. This equation is obtained by averaging (1), which gives (Mofffat 1978, Parker 1979)

$$\partial_t \overline{\mathbf{B}} = \nabla \times \langle \mathbf{v} \times \mathbf{b} \rangle + \nabla \times (\overline{\mathbf{V}} \times \overline{\mathbf{B}}) - \nabla \times \lambda \nabla \overline{\mathbf{B}}. \qquad (13)$$

When the turbulence is assumed to be isotropic, three dimensional, and incompressible, and $\langle \mathbf{v} \times \mathbf{b} \rangle$ is time independent, mean field theory with the minimal τ closure (Blackman

and Field 2002) gives

$$\langle \mathbf{v} \times \mathbf{b} \rangle \simeq \tau\tilde{\alpha}\overline{\mathbf{B}} - \tau\tilde{\beta}\nabla \times \overline{\mathbf{B}} + f(\overline{\mathbf{V}}), \tag{14}$$

where $\tau \sim \frac{1}{v_0 k_{0,v}}$ is a non-linear turbulent damping time, and in 3-D, $\tau\tilde{\beta} \lesssim \langle v^2\tau \rangle$ acts as a diffusion coefficient. The $\tilde{\alpha}$ term, important for the helical dynamo (Mofffat 1978, Parker 1979, Brandenburg and Subramanian 2005) survives only when conditions (such as rotation + stratification) are able to sustain a mean pseudoscalar. The last term in (14) schematically indicates additional terms that are a function of $\overline{\mathbf{V}}$.

Here we do not discuss the precise value of $\tilde{\alpha}$ and $\tilde{\beta}$ in (14), rather, we simply want to emphasize that these terms come from turbulence and would be absent without it. In the simplest case for which the second and fourth terms of (14) vanish but the third term remains, we can define the effective magnetic Reynolds number for the mean field equation (13) as $\tilde{R}_{M,e} = \frac{|\nabla\times(\overline{\mathbf{V}}\times\overline{\mathbf{B}})|}{|\nabla\times(\tau\tilde{\beta}+\lambda)(\nabla\times\overline{\mathbf{B}})|}$. In terms of the velocity spectrum, if $\lambda << \tau\tilde{\beta}$, we can approximate the total diffusion by the latter and bound it by its upper limit such that

$$\tau\tilde{\beta} + \lambda \sim \tau\tilde{\beta} \lesssim \langle v^2\tau \rangle \sim \frac{v_0}{k_{0,v}} \int_1^{k_\nu/k_{0,v}} \kappa^{-(q+3)/2} d\kappa. \tag{15}$$

For $q = 5/3$, $\tau\tilde{\beta} \sim v_0/k_{0,v}$. This highlights that in the presence of turbulence, the mean field can diffuse at a rate determined by a macroscopic turbulent diffusion coefficient rather than the microphysical diffusion coefficient. That is,

$$\tilde{R}_{M,e} \leq \frac{\overline{V}L}{\tau\tilde{\beta}} = \frac{\overline{V}}{Lk_{0,v}v_0}, \tag{16}$$

which can be unity for $\overline{V} \sim v_0$ and $Lk_{0,v} \sim 1$.

We emphasize again that $R_{M,e}$ of the previous section has a different definition from $\tilde{R}_{M,e}$ just defined. The former measures the ratio of advection terms to the microphysical dissipation terms in the total energy equation, while the latter applies only to the mean field equation and depends on our having introduced a macroscopic turbulent diffusivity $\tau\tilde{\beta}$ to replace some triple fluctuation terms within the advection term by a diffusion term. The total mean field diffusion coefficient is sum of turbulent and microphysical diffusivities and so the denominator of $\tilde{R}_{M,e}$ also includes contributions from both. When the turbulent diffusivity greatly exceeds the microphysical diffusivity, the the denominator of $\tilde{R}_{M,e}$ is dominated by the former. This contrasts the denominator of $R_{M,e}$ which only involves microphysical dissipation. The reason that $R_{M,e}$ can also be small, as discussed in the previous section, is that small scale structures lead to a substantial microphysical dissipation term even if the microphysical diffusion coefficient is small.

That $\tilde{R}_{M,e}$ and $R_{M,e}$ can be small and independent of R_M results because turbulence cascades large scale structures down to microphysical scales where dissipation is fast. For

driven turbulence, the overall rate at which the cascade occurs is fixed by the forcing. As long as there is some microphysical mechanism of energy drain at small scales the rate of that mechanism does not strongly influence the overall rate of total energy dissipation; the system will develop the structures needed to accommodate the cascade rate.

The extent to which mean field turbulent diffusion in MHD is suppressed below the maximum kinematic macroscopic value by the backreaction from fluctuating and mean magnetic fields has been a topic of study (Vainshtein and Cattaneo 1992, Cattaneo 1994, Blackman 1996, , Rogachevskii and Kleeorin 2001, Blackman and Brandenburg 2002, Brandenburg and Subramanian 2005). Part of the reason for some apparent disagreement is that different researchers have made different simplifying assumptions (boundary conditions, steady vs. evolving states, 2-D vs. 3-D) and although each may present a result consistent with their assumptions the universality of the conclusions still needs to be sorted out. The suppression seems to be less dramatic in 3-D. More simulations are needed and further detailed discussion is beyond the scope of the present paper. To accommodate a range of probabilities we have written an inequality in (16), but the basic principles discussed as to why a cascade would dictate $R_{M,e} << 1$ and $\tilde{R}_{M,e} < R_M$ remain.

As discussed in Sec 1., the principles regarding the importance of the dissipation terms derive from, and are analogous to similar circumstances in the purely hydrodynamic case, with analogous effective dissipation numbers defined appropriately. In the absence of magnetic fields however, there is no backreaction so the hydrodynamic evolution of the mean velocity field and its turbulent diffusion are not subject to the issues raised in the previous paragraph.

4. Conclusions

We have calculated a dimensionless measure of the relative importance of advective vs. dissipative terms for the total magnetic energy equation in MHD that applies for arbitrary magnetic and kinetic spectra, and magnetic Prandtl number. We have discussed how the standard magnetic Reynolds number R_M defined using the largest gradient scale of a system does not accurately approximate this ratio for turbulent systems, even if the magnetic energy is dominated by large scale structures. Instead, this ratio is best approximated by $R_{M,e}$, a quantity largely independent of R_M for a range of turbulent spectra (though reverting to R_M for a laminar flow) and typically satisfies $R_{M,e} \sim O(1) << R_M$. The latter results because small scale structures augment the relative importance of the microphysical dissipation terms for a turbulent system compared to a laminar system with the same microphysical diffusion coefficient. In a steady state, such as those reached when MHD

turbulence is steadily forced, $R_{M,e} \sim 1$.

We have also discussed how the effective dissipation number $\tilde{R}_{M,e}$ for the mean magnetic field equation is defined differently from $R_{M,e}$ because the mean field equation picks out the gradient scale of the mean field only. A low value of $\tilde{R}_{M,e} << R_M$ results not from small scale gradients, but because macroscopic turbulent diffusion coefficients exceed microscopic diffusivities in closures that approximate nonlinear cascade terms by diffusion.

While the results that $\tilde{R}_{M,e} << R_M$, $R_{M,e} << R_M$, are not at all surprising when interpreting them analogously to steady-state Kolmogorov hydrodynamic turbulence, they do highlight the much greater role of dissipation in large R_M turbulent MHD flows compared to large R_M laminar flows. In particular, an important implication of $\tilde{R}_{M,e} << R_M$, $R_{M,e} << R_M$, and the very weak dependence of $R_{M,e}$ and $\tilde{R}_{M,e}$ on R_M is that the rate at which large scale MHD energy is dissipated can be much less sensitive to the actual microphysical dissipation mechanism in turbulent flows when compared to laminar flows. On a macroscopic dynamical time, the MHD energy cascades to scales small enough such that it can be drained at multiple sites. If the turbulence is steadily forced, then the system adjusts to produce enough small scale structures to accommodate the driving energy input rate. The overall energy dissipation rate can be insensitive to the microphysical reconnection (or other microphysical energy conversion rate) at any one site. Instead, the aggregate of dissipation sites emerges to drain the total magnetic energy on macroscopic dynamical time scales. Since many astrophysical settings are MHD turbulent, this highlights the importance of being precise when asking and studying the question "Is magnetic reconnection fast in astrophysics?"

The principle that small scale structures increase the effective reconnection rate is consistent with rapid reconnection mechanisms in the MHD models of Hendrix et al. (1996), Galasgaard & Nordlund (1996), Lazarian & Vishniac (1999), and in the rapid reconnection phase of Fan et al. (2004). Reconnection studies that start with a large scale laminar field and reconnect on collisionless scales (e.g. Birn et al. 2001, Shay et al. 2007) address a different regime. The latter can however apply to each of the small scale multiple dissipation sites within a globally turbulent MHD system. In some systems, collisionless individual structures are observationally resolved (e.g. the Earth's magnetosphere) and studying the plasma instabilities and plasma turbulence for different conditions on such scales is needed to understand when reconnecting structures incur a reconnection rate that depends only on its macroscopic properties (Kulsrud 2001). That being said, if a stellar or accretion disk corona is considered as a global entity, its overall energy dissipation rate in a steady state is determined by the dynamical rate of magnetohydrodynamic energy injection from below; the small scale structures develop to accommodate this rate. The overall dissipation rate

is best characterized by $\tilde{R}_M << R_M$ and $R_{M,e} << R_M$ even if the rate of each individual dissipation site were to depend on R_M.

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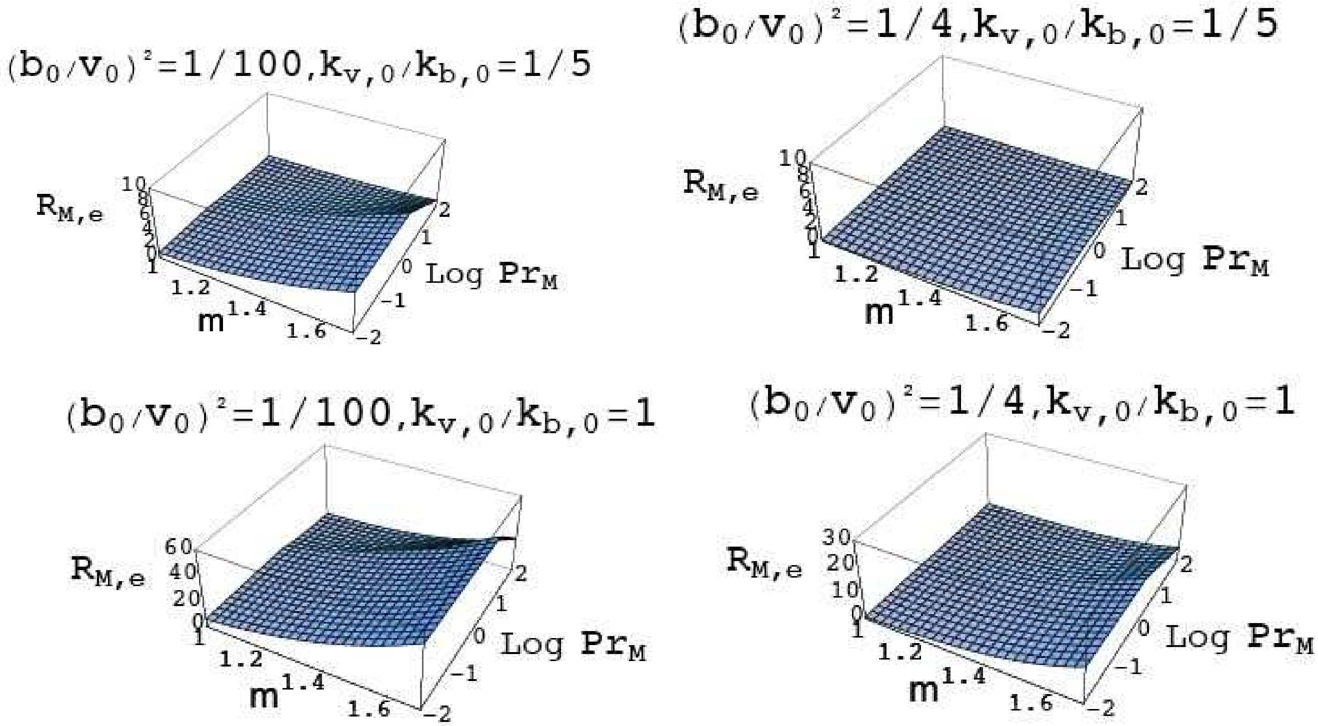

Figure 1: $R_{M,e}$ vs. magnetic spectral index m and magnetic Prandtl number Pr_M for $R_M = 10^3$. a) $(b_0/v_0)^2 = 1/100$, $k_{v,0}/k_{b,0} = 1/5$ b) $(b_0/v_0)^2 = 1/4$, $k_{v,0}/k_{b,0} = 1/5$ c) $(b_0/v_0)^2 = 1/100$, $k_{v,0}/k_{b,0} = 1$ d) $(b_0/v_0)^2 = 1/4$, $k_{v,0}/k_{b,0} = 1$ Note the weak dependence of $R_{M,e}$ on R_M.

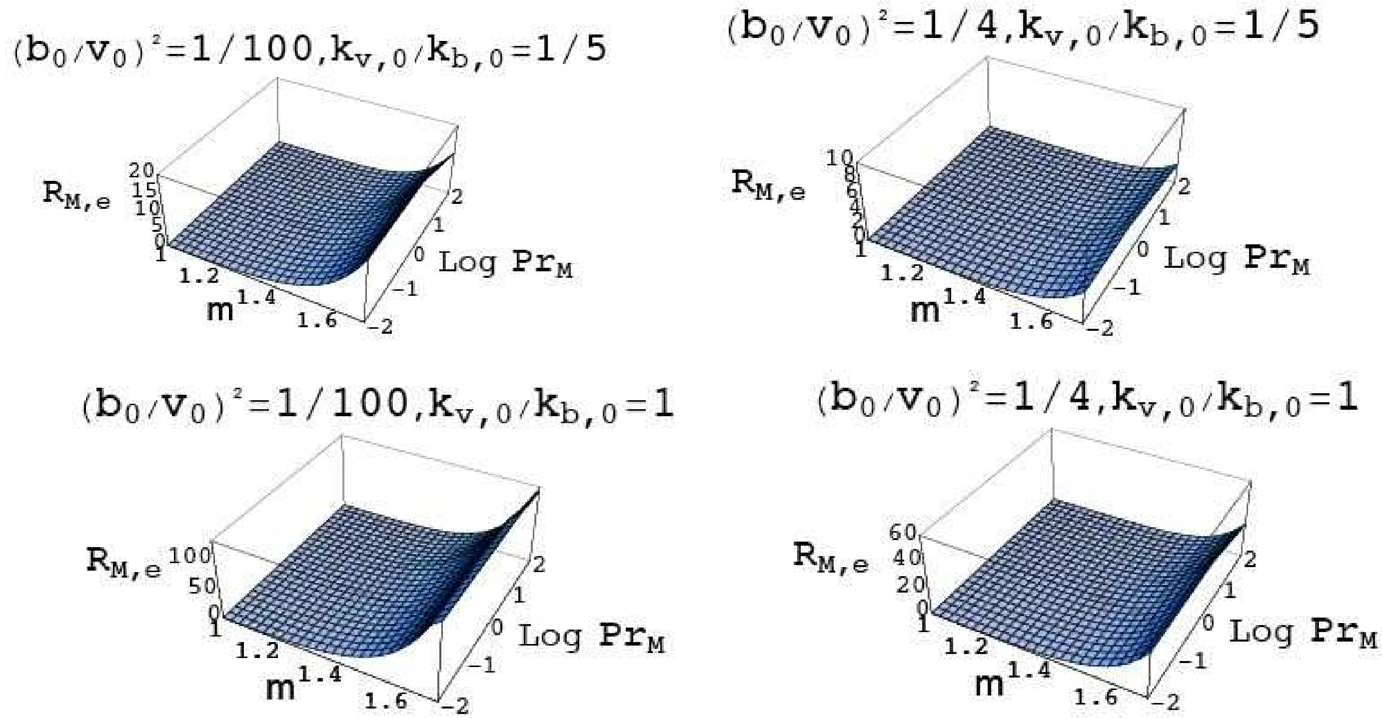

Figure 2: Same as Fig. 1 but with $R_M = 10^6$